# Engr 443

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***ADVANCED CONTROL***

***THEORY & DESIGN***

## U.S. AIR FORCE ACADEMY

Fall 2024

**Engineering 443 Course Syllabus**

LSN TOPIC READING\* DUE (HW & SR not collected)

**STATE-SPACE FUNDAMENTALS**

1 State-Space Representations: 2 *Review* Examples 2B, 2D,

Continuous and 2F (in text)

2 State-Space Representations: None

Discrete

3 Canonical Forms 3.6-3.7

4 Eigen-Decomposition A.8 3.1

5 Similarity Transformations A.8 4.1, Skills Review

6 Analytic eAt without Laplace/ 5.4, A.8

Cayley-Hamilton Theorem

7 Solution to the State Equations 3.1-3.4 6.1

**STATE-FEEDBACK CONTROL**

8 Controllability and Observability 5 7.1

9 State-Feedback Control 6.1-6.2 8.1-8.4

10 State-Feedback Control 6.3-6.5 9.1-9.2

11 State-Feedback Control 10.1-10.3

12 Observers 7.1-7.2 11.1-11.3

13 Observers 7.3 12.1-12.2

14 Combined Observer/State-Feedback 8.1-8.2 13.1

Control

15 **Graded Review 1** Review 14.1-14.2

16 **Lab 1** Lab 1 Write-up

**OPTIMAL CONTROL TECHNIQUES**

17 Optimization 9.1-9.3

18 Optimization - Equality Constraints 9.4 17.1-17.4

19 Optimization - Continuous Time None 18.1, **Lab 1**

LSN TOPIC READING\* WORK DUE

20 Linear Quadratic Control 9.8 19.1

21 Linear Quadratic Example 9.5 20.1

22 Linear Quadratic with Feedback 9.5

23 Linear Quadratic Regulator (LQR) None 22.1

24 LQR - Symmetric Root Locus None 23.1-23.2

25 LQR - MacFarlane/Potter None 24.1

26 In-Class LQR Demo None 25.1

27 Optimization Summary None 26.1-26.2

28 Discrete-Time Systems & Sampling None

29 Discrete-Quadratic Control None 28.1-28.2

30 **Lab 2** Lab Write-up 29.1-29.2

31 **Lab 2** Lab Write-up

32 **Graded Review 2** Review

**OPTIMAL STATE ESTIMATION**

33 Optimal State Estimation: “ Poor Man’s Explanation

Batch Least Squares of Kalman Filtering,” pp 1-9

(to “Application of...” on p 9)

34 Batch Weighted Least Squares None **Lab 2**

35 Sequential Batch Weighted Least Squares “Poor Man’s Explanation

of Kalman Filtering,” pp 9-24

34.1-34.2

36 Kalman Filtering 11.1-11.3 35.1

37 Kalman Filtering None

38 **Lab 3** Lab Write-up 37.1

39 **Lab 3** Lab Write-up

40 **Lab 3** Lab Write-up **Lab 3**

\* - CONTROL SYSTEM DESIGN, by Bernard Friedland

ENGR 443: ADVANCED CONTROL THEORY & DESIGN

**Course Goal:** In this course, we will extend the classical control tools learned in Linear Control Systems Analysis and Design, so that upon successful completion, you will be able to apply continuous and discrete (digital computer) modern control and estimation methods to analog (real world, continuous time) systems to meet stability, speed, and accuracy requirements.

**Course Objectives:** By the end of this course you will be able to:

- Model, transform, and solve continuous and sampled (analog system output) systems and reconstructed (digital controller output) signals in state-space.

- Analyze continuous and discrete state-space systems for stability, controllability and observability.

- Design continuous and discrete, optimal and non-optimal, state estimators and state feedback control systems which meet stability, speed, and accuracy requirements.

- Implement and show understanding of Kalman filters for use in aerospace problems.

1. Many experts believe that the rapid development of the aerospace industry in the 1960s was in large part due to the corresponding development of what is now called 'modern' control theory. Classical control, with its foundations rooted in the Laplace transform and the frequency response, provides the basis for understanding the general behavior of physical systems. But classical techniques, though effective for low order approximate models of single-input-single-output systems, were found to be cumbersome in dealing with complicated and multivariable systems. Aerospace problems, in particular those dealing with spacecraft control, yield very precise mathematical models for the process. By casting such models into what is now called the *state-space* framework, the powerful tools of linear algebra can then be applied to solve a wide range of control problems. This forms the basis for modern control.

2. Modern control theory provides engineers with powerful design methods; methods that are able to cope with higher system order and multiple inputs and outputs. It must be understood, however, that modern control theory has its own share of drawbacks. In Engr 443, we'll learn about both the power and the limitations of modern control theory. On the way, we'll highlight the interesting relationships that exist between classical and modern control.

3. Practical application of theory through computer simulation will form the cornerstone of your learning experience. I'm sure you'll find the course challenging, and I sincerely hope you find it enjoyable and rewarding as well.

4. Course Structure: The course is divided into four blocks of instruction as follows:

Block I State-Space Fundamentals

Block II State-Feedback Control

Block III Optimal Control Techniques

Block IV Optimal State Estimation

5. Textbook: The primary text for the course is Control System Design: An Introduction to State-Space Methods, by Bernard Friedland.

6. Grading: The total course weighting breakdown is as follows:

GR's (2 equally weighted) 36% \*

Formal Lab Project 1 10%

Formal Lab Project 2 10%

Formal Lab Project 3 14%

Final Exam 30% \*

Total 100%

**\* - Counts toward individual effort grade**

Your grade in this course will be determined based strictly on percentages. Your letter grade will be determined based on the following table.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | A- | B+ | B | B- | C+ | C | C- | D | F |
| >90% | >87% | >83% | >80% | >77% | >73% | >70% | >67% | >60% | <60% |

7. Skills Review and Homework: The skills review has problems that you should be able to do at the beginning of the course. The homework problems are designed to reinforce concepts presented in class and to further your understanding of those concepts. The problems were selected to accomplish these goals in minimum time. I will provide complete solutions to the skills review and homework, and you need to know how to do these problems, but they will not be collected.

8. Graded Reviews & Final Exam: All scheduled graded reviews are mandatory. If you will be unable to take an exam at the scheduled time, make prior arrangements with me to take it at a mutually-agreeable time and place. If you are ill on the day of an exam, contact me directly or have someone contact me as soon as possible.

9. Course Projects: Course projects include formal labs and computer projects. All course projects are team-effort and are mandatory. They are due at the beginning of class on the lesson indicated in the syllabus – one turn-in per team. Any late submissions will be graded in accordance with Astro Department policies. If you will not be in class on the day that a project is due, make prior arrangements to turn your project in on time.

10. Extra Instruction: If you need further help with material you don't understand, don't put off seeking help. I will be happy to provide extra instruction (drop-ins are OK but an appointment will ensure I’m available). Here’s how you can reach me:

Dr. Rob Brown Office: 6H-206 Phone: 719-930-4338 (or email or text via Teams)

11. Additional Guidelines: Overall course policies, including those not explicitly addressed here, adhere to the Department of Astronautics course policies. You are responsible for knowing them.



ROBERT B. BROWN, PhD

Engr 443 Course Director

**Engr 443 - Lab #1**

**State Feedback Control & Observers**

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| **This assignment is TEAM EFFORT.  You must complete your assignment with your assigned partner to submit for grading.  DO NOT copy anyone other group’s work or work with any cadet other than your partner on this assignment.**    **AUTHORIZED RESOURCES:  Any current Engr 443 instructor, course notes, course text. You may use generative AI programs like Chat GPT but must document in accordance with the department guidelines.** |

**Formal Lab Report (10%):**  Due Lesson 19

Objective: The objective of this lab is to investigate the behavior of a *continuous-time* full-state feedback control system, first using full state measurement, and then using only partial state measurement and a full-state observer. You will be designing a state-feedback controller for the “short-period” pitch rate behavior of a Boeing 787 Dreamliner subjected to a wind gust.

Overview: This lab will implement a feedback compensator using full state measurement and then using partial state measurement. An observer will be implemented to provide a state estimate for the partial state measurement case.

In-Lab Requirements: The “short-period” pitch rate behavior of a Boeing 787 can be represented by following system model. Implement the following system plant model in Simulink:



where 

and:

for full-state measurements (see Task I below)

C = 

and for partial-state measurements (see Task II below)

C = 

Task 1a: Full-state feedback using full state measurement in Simulink.

* Implement a *continuous-time* full-state feedback compensator so the compensated system has a damping ratio of  and a natural frequency of wn = 2.
* Exercise the analog compensated system with zero initial conditions and a pulse input of magnitude 1 and duration 0.5 sec. (The pulse will represent the wind gust.)
* Plot the history of the states during and after the wind gust.

Task 1b: Validate task 1a results using Matlab’s “lsim” command. (Use Matlab help.)

Task 2: Full-state feedback using partial-state measurements and a full-state observer in Simulink. (Use the same full-state feedback gains found in Task 1.)

- Using partial-state measurement, implement a full-state observer/full-state compensator.

To see the effect of observer pole placement, run the experiment for each of the following observer pole placement schemes. (Use zero initial conditions for the plant and for the I.C.’s of the observer, and a pulse input of magnitude 1 and duration 0.5 sec.)

1. Both observer poles at s = -2

2. Both observer poles at s = -4

3. Both observer poles at s = -8

- Plot the time histories of both the plant and observer outputs for each scheme.

**Lab Report Requirements**:

- The Astro Department’s Technical Report Writing Guide “Short Summary Report” Form

- Theoretical section should contain:

1. All derivations of compensators and observers.
2. Information needed to explain any expected lab observations.

- Your analysis section should include at least the following:

1. Comparisons between results of Tasks 1a, 1b and 2.

2. Any other analyses needed to justify your observations and conclusions.

**Engr 443 - Lab #2**

**Linear Quadratic Regulator**

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| **This assignment is TEAM EFFORT.  You must complete your assignment with your assigned partner to submit for grading.  DO NOT copy anyone other group’s work or work with any cadet other than your partner on this assignment.**    **AUTHORIZED RESOURCES:  Any current Engr 443 instructor, course notes, course text.  You may use generative AI programs like Chat GPT but must document in accordance with the department guidelines.** |

**Formal Lab Report (10%):**  Due Lesson 34

**Objective**: The objective of this lab is to design and evaluate a LQ regulator and a LQ controller with a given end time for a single-axis spacecraft attitude control system using optimal control theory.

**Overview:** This lab will implement a full state linear quadratic feedback for both cases.

**In-Lab Requirements**:

Task 1) Implement the following spacecraft model in Simulink.

Task 2) Using full state measurements and linear quadratic regulator feedback compensation, compute the feedback control gain, K, and implement your regulator in Simulink for the continuous system **given a unit step input**. Determine an appropriate stop time of the simulation. The LQ weighting matrices are:

 & R = 1

Task 3) Now, using full state measurements and linear quadratic controller with an end time of 30 seconds, compute the feedback control gain, K(t), and implement your controller in Simulink for the continuous system **given a unit step input**. The LQ weighting matrices are:

 ; R = 1; and 

The supplied Matlab function matint.m may be helpful to you for integrating the Riccati matrices. (Note: Even though you will need to integrate backward from P(tf) to P(0), matint.m will return a structure with P(t) values ordered from 0 to tf if you wish to use that as input for your Simulink model.)

Lab Report Requirements:

- The Astro Department’s Technical Report Writing Guide “Short Summary Report” Form

- Theoretical section should contain:

1. Discussion about the LQ regulator and controller. What are they, and how are they different from each other.

2. How do you compute the gains for the each of the above?

2. Brief estimate of what the expected output should be for each system.

- Your analysis section should include at least the following:

1. Plot the results of the states from Task 2.

2. A discussion and plots of the results of Task 2, including: Do the results make sense? Why did you pick the stop time that you did? What is the value of K?

3. A discussion and plots of the results of Task 3 and a comparison to the results of Task 2. (Include a plot of the P values for Task 3)

4. Any other analyses needed to justify you observations.

**Engr 443 - Lab #3**

**Kalman Filter for Gravity Gradient Oscillations**

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| **This assignment is TEAM EFFORT.  You must complete your assignment with your assigned partner to submit for grading.  DO NOT copy anyone other group’s work or work with any cadet other than your partner on this assignment.**    **AUTHORIZED RESOURCES:  Any current Engr 443 instructor, course notes, course text.  You may use generative AI programs like Chat GPT but must document in accordance with the department guidelines.** |

**Formal Lab Report Due (14%)**: Lesson 40

**Objective**: The objective of this lab is to design and evaluate a continuous time Kalman Filter to estimate a satellite’s pitch angle relative to the local horizon.

**Overview:** A generic FalconSat has been launched and stabilized using a gravity gradient boom, although there are still pitch oscillations that have not been removed yet.

θ

A horizon sensor on the satellite allows us to measure the pitch angle θ but the sensor is very noisy and doesn’t give individual measurements that are accurate enough for the control system on board. What is needed is a Kalman Filter that incorporates those measurements real time and does a “best fit” to give a better estimate of the current pitch angle.

**In-Lab Requirements**: Develop a Simulink program that implements a continuous Kalman Filter that estimates the pitch angle. Our system can be modeled as:



where

 - modeling errors

 - measurement errors

Once we implement the Kalman Filter, our optimal observer will be modeled as:

 (note: assume no inputs, so we don’t need *B* or )



where





Gravity Gradient Pitch Equation (you will see this in AstroEngr 445)



where

 - pitch angle

 - mean motion

 - moment of inertia about the roll axis

 - moment of inertia about the yaw axis

 - moment of inertia about the pitch axis

The satellite oscillates according to this model and this is the model that we will use in the Simulink program. Keep in mind though that there are unmodeled perturbing forces in the real satellite (and in the plant that we will give you.)

States and Output

Define the state as 

The output, *y*, is the direct measurement of θ

FalconSat Data

Moments of Inertia

*Ir* = 67.40 kg m2

*Ip* = 67.45 kg m2

*Iy* = 1.31 kg m2

Orbit Characteristics

alt = 560 km

inclination = 0º

eccentricity = 0

Horizon Sensor Data

Measurement standard deviation = 5º

Lab Tasks:

1. Develop a Simulink model to implement a Kalman Filter to estimate the pitch angle at each point in time. A Simulink satellite plant block is provided for you in the course files folder. Set the Max step size to 1 sec in Simulink by going to “Simlulation/Configuration Parameters…” (or in MatLab 2019 and later: Modeling Tab… Model Settings…) and set the Stop time to 5000 sec. In that same window, set Relative tolerance to 1e-6.

2. Run the simulation using an initial value for Q of all zeros.

-Compare the Kalman Filter output to the raw measurements on the same plot. (A Mux block may be helpful here to combine , so they can be output to the same scope.)

-Plot the covariance matrix (P) values.

3. Adjust the “tuning” numbers in the Q matrix to and create the plots as in Task 2.

4. “Tune” the matrix yourself by adjusting the values in Q so that you get the “best” results. Then create plots as in Task 2.

Lab Report Requirements:

- Use the Astro Department’s Technical Report Writing Guide “Short Summary Report” Form

- Theoretical section should contain:

1. A complete description of the Kalman Filter and the associated equations.

2. The description of how you determined the values for the matrices, A, C and W.

- Your analysis section should include at least the following:

1. A discussion of the results of Task 2. Include observations of strong or weak points.

2. A discussion of the results of Task 3. Include observations of strong or weak points.

1. A discussion of the results of Task 4. Include observations of strong or weak points and why you picked the values of Q that you did.

Hints:

- In the Scope blocks you may need to open them; click on the Parameters icon; select the Data History tab; and uncheck the box that says “Limit data points…”

- The P matrix can be initialized with 10000 on the diagonal so that it indicates that we have no confidence in the initial guess of the state.

- If you use a User Defined Function block for computing, you may have to uncheck the “Collapse 2-D results to 1-D” block, under the block parameters.